

STATISTICAL ANALYSIS OF QUALITY OF SURFACE FINISH  
IN MILLING OPERATIONS

by

HUSAIN ALI HASHAM

L. M. E. (Mechanical Engineering), Government Polytechnic,  
Hyderabad, India, 1960  
B. S. (Industrial Engineering), Kansas State University, 1963

---

A MASTER'S THESIS

submitted in partial fulfillment of the  
requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY  
Manhattan, Kansas

1964

Approved by:

George F. Schrader  
Major Professor

LD  
2668  
T4  
1964  
H34  
C.2  
Document

## TABLE OF CONTENTS

INTRODUCTION . . . . .	1
Conventional Milling . . . . .	1
Climb Milling . . . . .	3
OBJECTIVE . . . . .	4
BASIC PROCEDURE . . . . .	4
DESIGN OF EXPERIMENT . . . . .	5
Statistical Methods . . . . .	5
Analysis of Variance . . . . .	6
TEST CONDITIONS . . . . .	7
SELECTION OF THE LEVELS . . . . .	9
Type of Milling . . . . .	9
Cutting Speed . . . . .	9
Rate of Feed . . . . .	9
REVIEW OF LITERATURE . . . . .	9
EXPERIMENTAL EQUIPMENT . . . . .	12
Profilometer Group III . . . . .	12
Work Piece . . . . .	14
Cutter . . . . .	14
FACTORIAL IN RANDOMIZED COMPLETE BLOCK DESIGN . . . . .	14
REPLICATION . . . . .	17
EXPERIMENTAL PROCEDURE . . . . .	19
MATHEMATICAL MODEL OF THE EXPERIMENT . . . . .	23
ASSUMPTIONS FOR THE MODEL AND TEST OF HYPOTHESIS . . . . .	24
CALCULATIONS . . . . .	25

RESULTS AND DISCUSSIONS	31
Two Factor Interaction	34
Multiple Comparison Test for Means	40
L S D Test	40
CONCLUSION	42
ACKNOWLEDGMENTS	43
REFERENCES	44

## INTRODUCTION

Two methods of removing metal are used in milling practice. One is called conventional milling or up milling; the other, climb milling or down milling.

### Conventional Milling

Milling machine operations have been performed by rotating the cutter in a direction opposite to that of the table movement or feed, as shown in Figure 1, in all but a few types of work since the milling process has been used in manufacturing. This method is known as conventional milling. For example, if the cutter turns in a clockwise direction, as viewed from the front, the work is fed to the cutter from the left to right, or against the cutter. The cutting edges are cutting upwards from the bottom of the cut to the top of the work, resulting in pressure against the movement of the work, the feed forcing the work against the cutter. This method of milling takes care of any "back lash" or "lost play" which exists between the table screw and the nut because the action of the cutter tends to force the work from the cutter.

In this method of milling, the cutter teeth come up from the bottom, and the chip, which is very thin at first, gradually increases in thickness as work feeds towards the cutter. Because of resistance to penetration, the actual cutting is delayed until some point A is reached as shown in Figure 1. The delayed cutting action is due to the cutter's sliding over or scraping the material to be cut until significant pressure has been built up to force the cutter to break through or bite into the surface of the material and produce a chip.

Fig. 1  
Conventional Milling

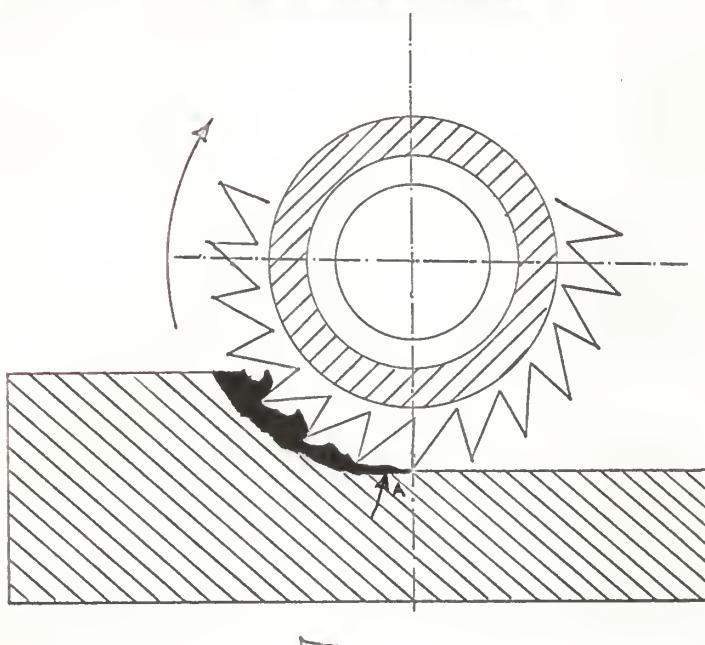
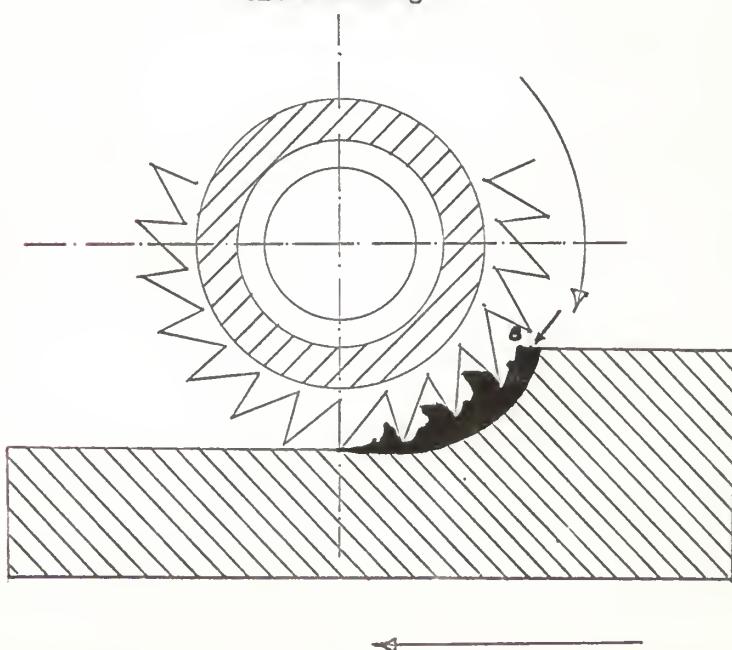


Fig. 2  
Climb Milling



Thus in conventional milling, the cutter continues alternately to slide over and to penetrate the surface of the material as the work is fed to the cutter. It is this cutter action which is responsible for the feed and revolution marks frequently seen on milled surfaces. This same action tends to dull the cutting edges.

#### Climb Milling

Climb milling or down milling differs from conventional milling in that the work is fed in the same direction as the movement of the cutter teeth, rather than against the movement of the teeth, as shown in Figure 2. In down milling the cutting edges are moving downwards from the top of the work to bottom of the cut. Climb milling exerts pressure down on the work and also tends to pull the work towards the cutter.

Back lash between the table screw and nut becomes troublesome with this method unless the machine is provided with some means of eliminating it. Some modern milling machines come equipped with a back lash eliminating device. This device overcomes the tendency of the cutter to climb on the work.

The chip formation during climb milling also differs from that of conventional milling. In its downward rotation the face of the tooth becomes almost parallel with the work as shown at B in Figure 2. The cutting edge moving from the top of the work through an arc to the bottom of the cut begins, contrary to up milling, with a chip of maximum thickness and permits a gradual disengagement of the teeth and the work. It follows then that in climb milling there is not the building up of pressure between the cutter and the work that occurs in conventional milling. Instead, there is a gradual diminishing of pressure which almost entirely eliminates feed marks and revolution marks from the milled surfaces.

## OBJECTIVE

The principal aim of this study was to determine which of these two types of milling, up or down, gives a better quality of surface finish or physical character of machined surface, under different machining conditions, i.e., cutting speeds and rate of feeds, on cold rolled mild steel SAE 1018. It would also be determined as to what amount of variability in the quality of surface finish is present between these two types of milling, up and down.

Statistical methods were employed to investigate the effects on surface finish characteristics of type of milling, up and down, with combinations of different levels of cutting speeds and rate of feeds.

## BASIC PROCEDURE

The experiment was performed with three factor factorial in a randomized complete block design, two types of milling, up and down, two levels of cutting speed and two levels of table feed. The number of replicates was chosen to be four. Other than changing the required type of milling, cutter speed and table feed care was taken to see that all relevant factors which may have an influence on the surface finish were either held constant or minimized as far as practically possible.

The mild steel work piece was cut in the form of a rectangle 6" x 3" in size, from the same stock of 3/4" thickness so as to have a more homogeneous experimental unit.

The whole experiment was performed by a high speed steel cutter of 4" in diameter, 22 teeth, 3° clearance angle, 10° rake angle and  $\frac{1}{2}$ " in width.

Before selecting the 3° clearance angle, a trial experiment was set up to check which clearance angle (eg:  $2\frac{1}{2}^{\circ}$ , 3°,  $3\frac{1}{2}^{\circ}$ ,  $4^{\circ}$ ) under the given constant

machining condition will give maximum number of samples of the same quality of surface finish before the cutter shows effect of wear.

This trial experiment on cutter for the clearance angle also disclosed the fact that the depth of cut of .030 of an inch did not wear out the cutter before giving 12 samples of some surface finish. It was, therefore, proposed to fix the cutter clearance angle at  $3^{\circ}$  and the depth of cut to .030 of an inch as fixed factors throughout the experiment along with other constant factors.

The measurement of surface finish was accomplished by means of profilometer Group III.

All machining operations were performed dry but air blast was applied for a check to see that the chips do not stick to the cutter.

All calculations pertaining to the statistical analysis of data were carried out on a Monroe desk calculator.

#### DESIGN OF EXPERIMENT

##### Statistical Method

Evaluating surface-finish testing methods offered several experimental problems. Machining data, even when reasonable precautions are taken, have a large variability. Therefore, to have an accurate indication of how any one variable affects surface finish many tests are required to offset the lack of control and resulting scatter in data. The task of examining many machining variables may be prohibitively expensive in time and material. Knowledge of how each variable independently affects surface finish will not tell how several variables together will affect surface finish.

The statistical technique of analysis of variance was chosen as the experimental method which would best fulfill the aims of this thesis.

### Analysis of Variance

Analysis of variance is a method by which the total variation in the data is separated into components in such a manner that their significance may be appraised. Components to which the variance is attributed may be individual factors or a combination of factors. When the variance associated with a component has been calculated, it is compared with the residual variance i.e., variance which is random in the experiment and cannot be ascribed to any component. Any component less than the .05 critical value (if .05 is chosen to be the critical value) is considered to be nonsignificant. This does not mean a variable not proved significant is insignificant (!). It does mean that the experiment under examination was not sensitive enough to reveal significance. It is possible for each tested variable to cause variation that is undetected by the analysis because other unaccounted-for (residual) variation may be large in comparison.

Residual or experimental variation, i.e., variation not attributable to a component under study, can be considered due to chance. In reality this variation is due to lack of complete control over the testing environment. Unknown variables and the inconvenience of holding all environmental conditions constant both cause residual variation. The experiments must be designed so that variations from these uncontrolled factors appears in the analysis as residual variation and is not attributed to a component. To accomplish the elimination of systematic variation caused by uncontrolled factors, all testing is run in a random order as determined by random number tables.

The greater the variation due to these uncontrolled factors, the less sensitive will be the results. Therefore, familiarity with the type of experiment is necessary for the investigator to include enough tests for the required sensitivity. If it is found that residual variation is a very small

percentage of total variation, the number of tests may have been greater than required. The necessary accuracy of the results and the cost of additional tests should both be considered when an experiment is designed.

There are several disadvantages in the use of analysis of variance. Results are not in curve form (2) and the mathematical relationship between variables is not typically determined. The only information obtained is the degree of certainty that the variable or combination of variables under study has some effect on results. In addition, preparation of the results requires knowledge of the technique and increased computational labor.

Snedecor's variance ratio, or F-test, will reveal if non-equal variances are significantly different only because of random or uncontrolled variation in the data and that due to sampling error.

To test for a significant difference in treatment mean effects the hypothesis is made that there is no difference in treatment means. This null hypothesis is tested by means of tables (3) of percentage points of the F distribution. These tables show for a given ratio of variance estimates and given degrees of freedom the probability of error if the hypothesis is rejected.

#### TEST CONDITIONS

As mentioned earlier, the principal aim of this thesis is to determine which of the two types of milling, up or down, with different combinations of other variables would give a better quality of surface finish (which is the characteristic to be measured). Therefore, the type of milling becomes one of the factors of major importance in the study with its two levels i.e., up milling and down milling.

Other factors that affect the surface finish are numerous. For example, cutting speed, rate of feed, cutter's clearance angle and rake angle, depth of cut, dry or wet machining, etc.

It was considered appropriate to limit the controlling factors to cutting speed and rate of feed other than the type of milling for investigation because of possible significance and ease of control. Therefore, the three factors considered are:

1. Type of milling.
2. Cutting speed in feet per minute.
3. Rate of feed in inches per minute.

The cutter wearout was of greater concern. This was one of the variables which would change from sample to sample. It was necessary to keep this variable under statistical control.

A trial experiment was conducted on a 6" x 3" section of SAE 1018 cold rolled mild steel with the combination of two types of milling with different levels of speeds and feeds to check how many samples these combinations would produce before the cutter shows significant signs of wear. All the trial tests were performed with the test cutter. In other words, how many samples would be considered statistically as one sample, working under different combinations of type of milling, with different levels of speeds and feeds. The different levels of speeds and feeds chosen for the trial tests were: Speeds: 79 feet per minute, 100 feet per minute, and 126 feet per minute. Feeds:  $1\frac{1}{2}$  inches per minute, 2 inches per minute, and 3 inches per minute.

It was observed in these trial experiments that with a depth of cut of .030" and clearance angle of  $3^{\circ}$  (which are constants; determined from previous trials) none of the milling speed and feed combinations at different levels give more than fourteen and not less than ten samples which would be considered statistically as one sample.

It was decided to use  $2^3$  factorial experiment where: power 3 stands for the three factors under consideration i.e., type of milling, cutting speed and rate of feed, and 2 stands for two levels of each factor. Therefore, the factorial experiment is of  $2^n$  series.

#### SELECTION OF THE LEVELS

##### Type of Milling

The two types of milling i.e., Conventional milling or Up milling and Climb milling or Down milling are designated as  $M_1$  and  $M_2$  respectively as the two levels of type of milling.

##### Cutting Speed

The two levels of speed selected are as follows:

$S_1$  = 79 feet per minute.

$S_2$  = 126 feet per minute.

##### Rate of Feed

The two levels of feed selected are as follows:

$F_1$  =  $1\frac{1}{4}$  inches per minute.

$F_2$  =  $3\frac{1}{4}$  inches per minute.

#### REVIEW OF LITERATURE

Little work has been done in laboratories to determine which of the two types of milling i.e., up or down, with various combinations of cutting speed and rate of feed, gives a better quality of surface finish. Cincinnati Milling Machine Company (4) has done some work in studying the surface milled by these

two types of milling but it was not determined as to what amount of variability in the quality of surface finish is present. It was also not determined whether the combination of different speeds and feeds with the two types of milling will alter the decisions (results).

The quality of finish or physical character of a machined surface is determined by these factors:

1. Tooth marks and revolution marks which depend on the geometry of the machining process used. These determine the 'waviness' of the surface.

2. Irregular marks of microscopic dimensions which result from the plastic flow of the material removed from the work piece in the form of chips. These determine the 'roughness' of the surface.

A milled surface is the result of the combination of innumerable elemental surfaces generated by individual cutter teeth. In forming this surface, each tooth leaves a mark on the final surface which is known as a tooth mark.

In peripheral milling, tooth marks characteristically lie approximately parallel to the axis of the cutter and are spaced at a distance equal to the feed per tooth. The mark of each individual tooth is not visible to the naked eye, but becomes distinct at higher feed per tooth.

In up milling, the tooth marks are produced by the direct action of the cutting edge of each tooth as it engages the surface generated by the cutting edge of the previous tooth.

In down milling, similar marks may result from the intersection of the plain of shear with the surface milled by the previous tooth. Owing to the nearly tangential direction in which the material yields at the end of tooth engagement, tooth marks usually are not obtained in down milling and the finished surface has a dull appearance free from the characteristic parallel markings which are present on a surface produced in up milling.

In addition to the tooth marks, a surface milled with peripheral milling cutter may show periodic variations having a wavy appearance and recurring with the frequency of the cutter revolution per minute. These marks are known as revolution marks.

Revolution marks are usually seen on surfaces milled in up milling. In down milling marks may not be visible due to the yielding of the material of the chip in an almost tangential direction as the tooth approaches the finished surface of the work.

The formation of the chip, which involves the plastic deformation of the material being removed from the work piece, affects the texture of the milled surface. At the beginning of contact of the cutting edge of a tooth with the work material, and for a short distance thereafter, the milled surface is usually smooth and has a uniformly shiny appearance. As the cutting action continues, the quality of the surface changes in accordance with the type of chip produced.

The type of chip commonly encountered, especially when milling steel with high speed steel cutters, is of continuous type where the frictional resistance to the flow of the chip along the face of the tool is so great as to cause formation of the built-up edge. With this type of a chip, the continuity of the machined surface is broken by highly deformed fragments of work material which have escaped from the lower part of the built-up edge. These fragments produce minute steps and cavities and thus give a rough and torn appearance to the finish of the machined surface.

## EXPERIMENTAL EQUIPMENT

The experiments were conducted on Cincinnati No. 1-18 Plain Automatic Milling Machines, 3 H.P. motor. It provided many different spindle speeds and table feeds. The lowest and highest spindle speed available was 50 r.p.m. and 1500 r.p.m. respectively. The lowest and highest table feeds was  $3/4"$  per minute and 30" per minute respectively. Micrometrical Manufacturing Company's Group III profilometer was used to determine the surface finish.

## Profilometer Group III

Figure 2. It is used in making surface roughness measurement. This is a direct reading instrument which measures average roughness height in micro-inches by passing a fine tracing point over the surface. The Group III combination includes the following:

The Type QC Amplimeter,

Type V Mototrace and BK Linkarm,

Type LK Tracer and the FT Skidmount.

Tracer converts the vertical movements of the tracing point into a small fluctuating voltage that is related to the height of the surface irregularity; a motor-driven device (Mototrace) for operating the traces, and the Amplimeter. The amplimeter receives the voltage from the traces, amplifies and integrates it, so that it may be read directly on the micro-inch meter or put in curve form on a recorder (not used in this experiment). The process is a continuous one, and the instrument shows the variation in average roughness from a reference line. Readings may be either arithmetical or root-mean-square (rms). The rms average is to be preferred as it gives more weight to the larger deviations from the reference line and is slightly larger than the arithmetical average.



Figure 3. Photograph showing the profilometer Group III and the mild steel work piece.

The Type V Mototrace can be used with all skid-type tracers, and provides a steady tracing speed of approximately 0.3" per second over any desired length of trace from 1/16" to 2 3/4".

The Type BK Linkarm provides the necessary linkage between the Type V Mototrace and the Type LK Tracer.

Profilometer Group III requires no sensitivity adjustment or calibration.

#### Work Piece

The work piece employed was 6" x 3" x 3/4" mild steel SAE 1018 cold finish. The work piece was held in milling machine vice.

#### Cutter

The cutter employed was high speed steel cutter of 4" in diameter,  $\frac{1}{2}$ " in width, 22 cutting edges, clearance angle of  $3^{\circ}$  and rake angle of  $10^{\circ}$ .

#### FACTORIAL IN RANDOMIZED COMPLETE BLOCK DESIGN

If the whole of the experimental material, area, or time is not homogeneous, it may be possible to stratify or group the material into homogeneous subgroups. This is one of the methods for controlling the variability of experimental material (5). If the treatments are applied to the relatively homogeneous material within each stratum or group and replicated on the other strata, the design is randomized complete block. For the completely randomized design, no stratification of the experimental site (space, material or time) is made. The treatments are randomly allotted to the block or work piece. In the randomized complete block design the treatments are randomly allotted within each stratum, i.e., the randomization is restricted. Also, the variation among strata (replicate or blocks) is removed from the variation

among replicates within treatments. Therefore, if it is desired to control one source of variation by stratification, the experimenter should select the randomized complete block rather than the completely randomized design.

The chief advantages of the randomized complete block design are:

1. Accuracy. This design has been shown to be more accurate than the completely randomized design for most types of experimental work. The elimination of the block sum of squares from the error sum of squares usually results in a decrease in the error mean square.

2. Flexibility. No restrictions are placed on the number of treatments or on the number of replicates in the experiment. In general, at least two replicates are required to obtain tests of significance. In addition, the check or other treatments may be included more than once with little complication to the analysis.

3. Ease of analysis. The statistical analysis is simple and rapid. Moreover, the error of any treatment comparison may be isolated and any number of treatments may be omitted from the analysis without complicating it. These facilities may be useful when certain treatment differences turn out to be very large, when some treatments produce failures, or when the experimental errors for the various comparisons are heterogeneous.

The factorial experiment, in itself, is not considered as an experimental design (6). Any of the experimental designs, e.g.; completely randomized, randomized complete, etc., may be used for the factorial experiment. The choice of treatments, the "treatment design", determines whether or not the experiment is a factorial.

Factorial in randomized complete block design is selected because it is best suited for our purpose. Prior to the experiment under study, it was not known whether or not the different lengths of mild steel plate would exhibit

the same machining properties, i.e., it was not known prior to the experiment whether the material was homogeneous or not.

A group of treatments which contains two or more levels of two or more factors or substances in all combinations is known as "factorial" arrangement.

In a factorial experiment all combinations are used to evaluate main effects and interactions. This is not true for other arrangements of treatments. Therefore, there is some loss in information in nonfactorial arrangements owing to the fact that only a fraction of the total number of experimental units is used to evaluate an effect. The percentage of the total number of experimental units used to evaluate an effect depends upon the composition of treatments in a nonfactorial experiment. In addition, the main effects in a factorial arrangement are evaluated over a wider range of conditions, and information is obtained on the interaction of factors. In fact, if the objective of the experiment is to estimate main effects and interactions, a factorial arrangement of treatments is optimum for this purpose.

The use of all experimental units in evaluating an effect increases the efficiency of the particular experiment. Fisher, R. A. (7) and Yates (8) explain that it is often desirable to include additional factors in an experiment in order to observe the effects of interest over a wider range of conditions.

Factorial experiments result in unbiased conclusions even if trends or gradients are present in the experiment, although their effectiveness is somewhat reduced. Worker fatigue, temperature, light, and humidity changes, gradients in material, certain types of interactions, etc., account for trends in experimental material.

The advantages of a factorial experiment may be summarized as follows:

1. All experimental units are utilized in evaluating effects, resulting in the most efficient use of resources.

2. The effects are evaluated over a wider range of conditions with the minimum outlay of resources.
3. An estimate of the interaction of the factors is obtainable.
4. Unbiased estimates of effects are obtained whether or not time trends are present.
5. A factorial set of treatments is optimum for estimating main effects and interactions.

#### REPLICATION.

Since variability is almost universal, replication or blocking (the repetition of the set of treatments in the experiment) is, or should be, practiced in nearly all experimental work. Fisher, R. A. (9) explains that the two conditions necessary to obtain a valid estimate of the experimental error are replication and randomization. Hence, replication is an important feature of experimental work.

It was decided to use four replicates or blocks.

Having decided upon two levels of speed, two levels of feed and two types of milling, up and down, there arise in all  $2 \times 2 \times 2 = 8$  different types of milling feed-speed combinations, or 8 different treatment possibilities within the scope of this experiment.

The randomized complete block technique requires complete randomization of the different combinations of levels of factors within each block. In order to assign these eight treatment combinations randomly on each block, Tables of Random Permutations (10), Permutations of 9, Table 1 was used.

No. 1 designated a machining combination  $M_1, S_1, F_1$  where  $M_1$  represents conventional or up milling,  $S_1$  a speed of 79 f.p.m. and  $F_1$  a feed of  $1\frac{1}{4}$  inches per minute.

## 15.5 Tables of Random Permutations

TABLE 15.6 PERMUTATIONS OF 9

5	5	6	7	1	4	3	3	7	3	8	7	4	6	3	9	7	4	9	4	9	2	2	8	8	2	7	9	3	5	8	3	1	9	4		
4	1	2	8	2	7	1	1	2	9	9	5	7	8	2	8	9	3	6	6	1	7	7	2	4	4	8	5	7	3	3	7	4	5	6		
9	3	3	2	9	8	8	8	4	5	2	4	6	1	0	5	0	7	7	8	7	4	4	7	1	7	3	2	8	6	6	1	2	2	2		
7	9	7	4	3	5	5	5	2	9	2	1	6	5	3	5	7	8	5	1	9	0	5	1	9	1	3	6	5	1	4	9	2	9	3	7	8
1	6	9	6	5	6	9	4	3	6	4	3	9	2	9	5	1	8	2	3	8	3	3	2	8	9	6	1	2	4	5	7	6	9			
6	4	4	3	6	2	4	6	8	1	7	9	3	4	1	6	2	6	4	2	5	2	9	8	5	9	9	2	4	2	8	9	6	9	8	1	
S	7	8	1	7	1	2	5	6	8	3	1	2	9	8	4	4	1	8	7	6	5	1	6	7	5	4	3	5	1	1	4	3	1	7		
3	2	1	9	4	3	6	7	5	7	6	8	8	7	7	2	5	9	5	1	3	8	5	4	6	3	6	7	9	4	5	2	5	4	5		
2	8	5	5	8	9	7	9	1	4	5	2	1	5	4	1	3	2	3	5	4	6	9	5	1	1	8	6	7	7	8	6	3	3			
7	4	6	1	5	9	2	2	2	9	2	8	1	7	3	2	4	2	1	9	2	4	8	3	1	2	6	5	4	8	8	4	9	4	2		
9	3	8	3	2	1	1	1	9	8	9	4	9	5	4	8	8	8	6	7	7	5	4	6	5	3	2	7	6	9	3	8	2	1			
1	6	3	4	7	6	5	8	4	5	6	1	7	1	9	5	2	6	3	5	5	6	9	8	1	3	6	7	9	7	7	9	7	9	7		
6	8	2	8	4	8	7	6	3	6	5	7	5	4	5	9	7	5	8	3	9	9	7	7	8	5	3	3	5	6	9	4	6	9			
4	1	4	7	8	2	3	9	3	4	4	2	2	3	6	4	7	4	2	5	6	3	3	6	9	1	7	8	5	4	4	5	2	1	4		
2	9	1	9	3	7	9	6	6	2	1	6	4	6	1	7	9	9	7	4	1	8	4	1	8	9	2	7	9	3	1	8	3	5	5		
5	5	5	5	1	3	7	4	7	7	8	5	8	9	2	1	5	1	3	2	9	6	2	8	4	2	8	4	1	9	5	7	1	3	3		
8	2	9	2	9	8	6	5	3	7	9	6	8	8	3	1	6	9	7	4	1	6	9	3	4	4	6	6	2	7	2	6	8	8			
3	7	7	6	6	5	4	3	1	1	3	3	2	7	6	3	7	3	4	1	3	2	1	2	2	7	1	4	2	7	2	1	5	7	6		
9	7	7	5	5	5	9	9	3	8	9	6	1	7	5	8	6	1	2	9	8	3	3	2	3	7	7	3	7	6	6	5	5	5			
3	8	1	7	2	6	2	7	1	6	4	1	3	4	2	3	6	2	4	3	2	8	6	1	2	7	8	9	7	4	7	7	4	7	7		
4	3	4	2	7	7	3	1	7	2	9	1	5	4	8	6	6	2	1	7	8	5	1	7	1	3	6	3	1	2	3	1	2	3	1		
5	9	2	8	3	7	5	8	9	8	9	2	1	7	1	7	1	2	3	8	3	4	9	9	7	7	8	5	1	7	1	7	8	5	1		
1	6	5	1	1	5	6	4	4	1	7	3	5	8	9	9	1	2	3	6	3	2	5	9	5	9	1	2	3	1	2	3	1	2	3		
6	2	8	3	6	2	5	5	2	2	6	8	9	1	7	5	6	4	7	4	6	4	1	7	4	6	4	1	2	8	8	6	6	5	9	1	
2	4	9	6	4	1	8	3	5	4	3	6	5	9	4	8	5	9	7	9	8	1	6	8	1	4	5	5	9	5	2	4	5	9	4		
8	5	6	9	9	2	5	2	6	7	6	8	7	8	3	9	1	9	4	2	5	6	4	7	4	5	2	3	2	8	2	3	3	2	3		
7	1	3	4	8	4	1	8	9	3	6	4	9	5	5	3	7	4	5	9	7	2	3	1	2	2	7	1	4	8	1	8	4	6	9	1	
7	4	9	8	7	9	7	1	7	1	9	2	3	8	7	7	3	5	5	1	6	4	9	7	8	6	1	8	2	9	7	4	3	4	7	7	
5	6	1	2	6	4	6	1	4	5	9	1	2	8	2	4	3	6	2	6	1	2	8	8	6	2	7	8	9	7	4	7	7	7	4	7	
4	9	3	5	6	1	1	8	4	8	3	5	4	9	3	3	6	1	2	3	2	6	8	7	7	4	5	3	8	5	8	5	9	5	1		
3	3	2	8	5	2	3	2	2	7	3	8	6	9	4	1	8	6	1	7	9	2	3	6	3	9	5	7	7	7	1	2	8	1	2		
2	1	4	9	4	4	6	2	8	3	2	7	6	5	1	5	7	3	1	2	9	8	4	1	3	6	3	1	2	9	9	1	5	7	7		
9	7	5	4	5	3	9	7	9	9	1	4	2	3	4	6	4	7	4	4	3	2	5	2	2	8	4	2	6	3	3	5	6	3	3		
6	2	6	3	9	8	8	5	5	5	8	7	7	2	9	3	4	5	8	9	4	8	2	4	9	4	4	8	1	2	9	9	1	2	9		
8	5	8	7	1	2	3	9	3	7	4	1	5	1	5	8	9	7	6	4	5	3	5	8	1	6	8	5	2	3	4	6	4	5	8		
1	8	7	6	3	7	5	4	6	6	6	8	9	4	6	1	2	2	9	9	6	4	1	8	5	2	7	9	3	6	7	7	2	9	6		
8	4	6	8	6	2	1	9	9	7	2	2	1	8	9	5	1	9	2	4	5	2	6	2	8	1	6	8	3	8	1	9	4	1			
9	9	4	5	8	4	4	8	7	8	8	7	5	9	7	3	6	4	7	7	3	8	5	3	6	4	4	6	7	7	6	6	8	7	8		
6	6	3	1	1	6	8	3	1	9	7	5	7	5	5	6	5	1	8	5	2	4	3	8	2	5	1	4	3	6	4	9	7	8	6		
7	3	7	7	2	7	3	6	2	2	3	8	9	4	6	4	7	2	6	9	7	9	7	4	1	3	8	2	6	5	3	1	4	1			
2	8	9	3	4	1	5	5	1	5	4	3	6	4	7	8	7	5	3	9	5	8	6	5	8	2	7	9	2	5	3	4	3	5			
3	7	2	6	9	8	6	4	6	3	4	1	9	6	4	8	4	7	2	1	3	6	3	5	5	1	2	2	6	9	9	9	9	9			
5	1	8	4	5	9	9	1	8	4	1	9	4	3	2	8	2	9	6	3	4	9	9	2	7	1	2	4	9	8	2	6	2				
4	5	5	2	7	3	2	7	3	6	9	3	2	1	8	9	7	7	2	1	6	9	7	7	9	5	9	1	8	7	7	1	5	7			
1	2	1	9	3	5	7	2	4	5	6	6	7	3	2	4	3	3	1	8	1	5	4	7	9	3	4	9	1	4	5	2	3				

Similarly, No. 2 designated a combination of  $M_1$ ,  $S_1$ ,  $F_2$  and so forth. Tables 2, 3, 4 and 5 show the designated machining combination and the result of selecting a column of 9 permutation. The 9th number was discarded as there were eight treatment combinations. Machining was then performed keeping strictly to the randomized pattern within each block.

#### EXPERIMENTAL PROCEDURE

The work piece of dimension 6" x 3" x 3/4" was held in the vice and checked for grip and level. The required speed, feed and type of milling was obtained by changing the required gears and direction of the cutter on the arbor. The depth of cut was fixed to .030". The setup is shown in Figure 4.

The cutter was resharpened after every 8 samples (one replicate), and care was taken to maintain  $3^\circ$  clearance angle on the cutter.

Air blast was used to check that the chips do not stick to the cutter.

After each replicate, the work piece was removed from the vice and spindle oil was applied on the samples to assure no rust forms on the samples before taking the measurement by the profilometer. It is to be added here that the measurement on all the four replicates were taken at one time, therefore, it was felt necessary to apply oil on the samples.

Before taking the measurement, the oil was blown from all the samples on the four replicates by the air blast.

After the experiment, the data on surface finish was collected by profilometer and is listed in Table 6.

Table 2. The sequence of machining conditions for replicate No. 1 under complete randomization.

Type of Milling	Speed	Feed
M <sub>1</sub>	S <sub>2</sub>	F <sub>1</sub>
M <sub>2</sub>	S <sub>2</sub>	F <sub>2</sub>
M <sub>2</sub>	S <sub>1</sub>	F <sub>1</sub>
M <sub>1</sub>	S <sub>2</sub>	F <sub>2</sub>
M <sub>2</sub>	S <sub>1</sub>	F <sub>2</sub>
M <sub>2</sub>	S <sub>2</sub>	F <sub>1</sub>
M <sub>1</sub>	S <sub>1</sub>	F <sub>1</sub>
M <sub>1</sub>	S <sub>1</sub>	F <sub>2</sub>

Table 3. The sequence of machining conditions for replicate No. 2 under complete randomization.

Type of Milling	Speed	Feed
M <sub>2</sub>	S <sub>1</sub>	F <sub>1</sub>
M <sub>1</sub>	S <sub>1</sub>	F <sub>2</sub>
M <sub>1</sub>	S <sub>2</sub>	F <sub>2</sub>
M <sub>2</sub>	S <sub>1</sub>	F <sub>2</sub>
M <sub>2</sub>	S <sub>2</sub>	F <sub>2</sub>
M <sub>1</sub>	S <sub>1</sub>	F <sub>1</sub>
M <sub>2</sub>	S <sub>2</sub>	F <sub>1</sub>
M <sub>1</sub>	S <sub>2</sub>	F <sub>1</sub>

M<sub>1</sub> = Conventional or Up Milling   S<sub>1</sub> = 79 ft. per minute   F<sub>1</sub> = 1 $\frac{1}{4}$  inches per minute

M<sub>2</sub> = Climb or Down Milling       S<sub>2</sub> = 126 ft. per minute   F<sub>2</sub> = 3 $\frac{1}{4}$  inches per minute

Table 4. The sequence of machining conditions for replicate No. 3 under complete randomization.

Type of Milling	Speed	Feed
M <sub>1</sub>	S <sub>1</sub>	F <sub>1</sub>
M <sub>1</sub>	S <sub>1</sub>	F <sub>2</sub>
M <sub>2</sub>	S <sub>2</sub>	F <sub>2</sub>
M <sub>1</sub>	S <sub>2</sub>	F <sub>2</sub>
M <sub>1</sub>	S <sub>2</sub>	F <sub>1</sub>
M <sub>2</sub>	S <sub>1</sub>	F <sub>2</sub>
M <sub>2</sub>	S <sub>2</sub>	F <sub>1</sub>
M <sub>2</sub>	S <sub>1</sub>	F <sub>1</sub>

Table 5. The sequence of machining conditions for replicate No. 4 under complete randomization.

Type of Milling	Speed	Feed
M <sub>2</sub>	S <sub>1</sub>	F <sub>1</sub>
M <sub>2</sub>	S <sub>2</sub>	F <sub>2</sub>
M <sub>1</sub>	S <sub>1</sub>	F <sub>2</sub>
M <sub>1</sub>	S <sub>1</sub>	F <sub>1</sub>
M <sub>1</sub>	S <sub>2</sub>	F <sub>2</sub>
M <sub>2</sub>	S <sub>1</sub>	F <sub>2</sub>
M <sub>1</sub>	S <sub>2</sub>	F <sub>1</sub>
M <sub>2</sub>	S <sub>2</sub>	F <sub>1</sub>

M<sub>1</sub> = Conventional or Up Milling   S<sub>1</sub> = 79 f.p.m.   F<sub>1</sub> = 1 $\frac{1}{4}$  inches per minute

M<sub>2</sub> = Climb or Down Milling   S<sub>2</sub> = 126 f.p.m.   F<sub>2</sub> = 3 $\frac{1}{4}$  inches per minute



Figure 4. Photograph showing the setup of milling machine and work piece.

Table 6. Surface finish obtained during the machining of mild steel SAE 1018 in a two factor factorial randomized complete block design.

Replicate 1				Replicate 2					
M <sub>1</sub>		M <sub>2</sub>		M <sub>1</sub>		M <sub>2</sub>			
S <sub>1</sub>	S <sub>2</sub>								
F <sub>1</sub>	18	20	70	60	F <sub>1</sub>	16	18	65	60
F <sub>2</sub>	30	20	80	55	F <sub>2</sub>	27	30	75	60

Replicate 3				Replicate 4					
M <sub>1</sub>		M <sub>2</sub>		M <sub>1</sub>		M <sub>2</sub>			
S <sub>1</sub>	S <sub>2</sub>								
F <sub>1</sub>	16	20	75	70	F <sub>1</sub>	18	20	70	65
F <sub>2</sub>	30	25	75	50	F <sub>2</sub>	32	25	80	55

#### MATHEMATICAL MODEL OF THE EXPERIMENT

The mathematical model developed for data obtained under three factor factorial is as follows:

$$\begin{aligned}
 x_{ijkl} = & M + R_i + A_j + B_j + C_k \\
 & + (AB)_{ij} + (AC)_{ik} + (BC)_{jk} \\
 & + (ABC)_{ijk} + E_{ijkl}
 \end{aligned}$$

where  $i$  = Type of milling =  $r$  = 2 types.

$j$  = Cutting speed =  $s$  = 2 levels.

$k$  = Rate of feed =  $t$  = 2 levels.

$l$  = Replicates =  $q$  = 4 numbers.

and

M = Mean effect

$R_1$  = Effect of the  $q^{\text{th}}$  replicate

$A_i$  = Effect of the  $i^{\text{th}}$  type of milling

$B_j$  = Effect of the  $j^{\text{th}}$  level of cutting speed

$C_k$  = Effect of the  $k^{\text{th}}$  level of rate of feed

$(AB)_{ij}$  = Two factor interaction effect of the  $i^{\text{th}}$  type of milling with the  $j^{\text{th}}$  level of cutting speed

$(AC)_{ik}$  = Two factor interaction effect of the  $i^{\text{th}}$  type of milling with the  $k^{\text{th}}$  level of rate of feed

$(BC)_{jk}$  = Two factor interaction effect of the  $j^{\text{th}}$  level of cutting speed with the  $k^{\text{th}}$  level of rate of feed

$(ABC)_{ijk}$  = Three factor interaction effect of the  $i^{\text{th}}$  type of milling with the  $j^{\text{th}}$  level of cutting speed and  $k^{\text{th}}$  level of rate of feed

$E_{ijkl}$  = Effect of the experimental unit in the  $l^{\text{th}}$  replicate to which the  $(ijk)^{\text{th}}$  treatment has been randomly assigned.

#### ASSUMPTIONS FOR THE MODEL AND TEST OF HYPOTHESIS

Certain statistical assumptions pertaining to fixed effects treatment designs are as follows:

1. If we assume that  $A_i$ ,  $B_j$ ,  $C_k$ ,  $(AB)_{ij}$ ,  $(AC)_{ik}$ ,  $(BC)_{jk}$  and  $(ABC)_{ijk}$  are finite populations composed of the particular levels used in the experiment, then

$$\sum_{i=1}^r A_i = \sum_{j=1}^s B_j = \sum_{k=1}^t C_k = \sum_{i \text{ or } j} AB_{ij} = \sum_{i \text{ or } k} AC_{ik} = \sum_{j \text{ or } k} BC = \sum_{i, j \text{ or } k} ABC = 0$$

2. The  $R_i$  are assumed to be normally independently distributed with a mean of zero and variance of  $\sigma_R^2$  i.e.,

$$R_i \text{ is NID } (0, \sigma_R^2)$$

3.  $E_{ijkl}$  the deviations due to unassignable causes are normally independently distributed with a mean of zero and variance  $\sigma_E^2$  i.e.,

$$E_{ijkl} \text{ is NID } (0, \sigma_E^2)$$

with these assumptions there would be the following null hypothesis for testing:

1. The surface finish is not significantly affected by the different types of milling, cutting speed and the rate of feed, i.e.,  $H_0: A_i = 0$ ,  $H_0: B_j = 0$  and  $H_0: C_k = 0$  respectively.

2. There exist no interaction effects between type of milling, cutting speed and rate of feed i.e., there exist no two factors or three factor interaction effects.

$$\text{i.e. } H_0: AB_{ij} = 0, H_0: AC_{ik} = 0, H_0: BC_{jk} = 0, H_0: ABC_{ijk} = 0$$

3. That there exist no difference between the blocks i.e.,  $H_0: \sigma_R^2 = 0$

Each of the above null hypothesis may be tested for significance by the F-test by dividing their respective mean squares by the experimental error mean square and comparing them with the critical ratios in the F-table at some chosen significance level at their appropriate degrees of freedom. The level of significance is maintained at .05 for all the tests.

#### CALCULATIONS

The data obtained on surface finish in Table 6, while machining under different machining conditions, is rearranged in Table 7. The interaction tables of type of milling x speed, type of milling x feed and speed x feed is obtained from Table 7 by adding the relevant values of surface finish common to type of milling, speed or feed appropriately as given in Tables 8, 9 and 10.

The calculations involved in obtaining the sums of squares and mean squares for the analysis of variance of a three-factor factorial in a randomized complete block design is as follows:

Table 7. Obtained from table 6.

		$M_1$		$M_2$		Feed Sum
		$S_1$		$S_1$	$S_2$	
$F_1$	18	20		70	60	
	16	18		65	60	
	16	20		75	70	
	<u>18</u>	<u>20</u>		<u>70</u>	<u>65</u>	
	68	78	146	280	255	535
$F_2$	30	20		80	55	681
	27	30		75	60	
	30	25		75	50	
	<u>32</u>	<u>25</u>		<u>80</u>	<u>55</u>	
Type Sum	119	100	219	310	220	<u>530</u>
			365			1065
						<u>749</u>
						1430

$$\text{Correction factor} = \frac{\sum_{i,j,k,l} x_{ijkl}^2}{rstq} = \frac{1430^2}{32} = 63,903 = C$$

1. Total corrected sum of squares ( $T_{cc}$ )

$$\begin{aligned}
 &= \sum_{i=1}^r \sum_{j=1}^s \sum_{k=1}^t \sum_{l=1}^q x_{ijkl}^2 - C, \text{ with } rstq - 1 \text{ degrees of freedom (d.f.)} \\
 &= 18^2 + 16^2 + 16^2 + \dots + 60^2 + 50^2 + 55^2 - 63,903 \\
 &= 80,962 - 63,903
 \end{aligned}$$

$$T_{cc} = 17,059 \text{ with 31 d.f.}$$

2. Replicate sum of squares corrected ( $R_{cc}$ )

$$\begin{aligned}
 &= \frac{\sum_{l=1}^q R_l}{rst} - C, \text{ with } q-1 \text{ d.f.} \\
 &= \frac{353^2 + 351^2 + 361^2 + 365^2}{8} - 63,903 \\
 &= 63,919 - 63,903
 \end{aligned}$$

$$R_{cc} = 16 \text{ with 3 d.f.}$$

## 3. Subclasses sum of squares corrected

$$\begin{aligned}
 &= \frac{\sum_{l=1}^q \left( \sum_{i=1}^r \sum_{j=1}^s \sum_{k=1}^t X_{ijkl} \right)^2}{q} - C \text{ with } rst - 1 \text{ d.f.} \\
 &= \frac{68^2 + 78^2 + 280^2 + \dots + 310^2 + 220^2}{4} - 63,903 \\
 &= 80,699 - 63,903 \\
 &= 16,796 \text{ with 7 d.f.}
 \end{aligned}$$

4. Experimental error sum of squares corrected ( $E_{cc}$ )

$$\begin{aligned}
 &= \text{Total sum of squares corrected} - \text{Subclasses sum of squares corrected} \\
 &\quad + \text{Replicate sum of squares corrected, with } (q-1)(rst-1) \text{ d.f.} \\
 &= 17,059 - 16,796 - 16 \\
 E_{cc} &= 247, \text{ with 21 d.f.}
 \end{aligned}$$

5. Type of milling sum of squares corrected ( $A_{cc}$ )

$$\begin{aligned}
 &= \frac{\sum_{l=1}^q \left( \sum_{i=1}^r \sum_{j=1}^s \sum_{k=1}^t X_{ijkl} \right)^2}{stq} - C, \text{ with } (r-1) \text{ d.f.} \\
 &= \frac{365^2 + 1,065^2}{16} - 63,903 \\
 &= 79,216 - 63,903
 \end{aligned}$$

$$A_{cc} = 15,313 \text{ with 1 d.f.}$$

6. Speed sum of squares corrected ( $B_{cc}$ )

$$\begin{aligned}
 &= \frac{\sum_{i=1}^s \left( \sum_{j=1}^r \sum_{k=1}^t \sum_{l=1}^q x_{ijkl} \right)^2}{rtq} - C, \text{ with } (s-1) \text{ d.f.} \\
 &= \frac{777^2 + 653^2}{16} - 63,903 \\
 &= 64,384 - 63,903 \\
 &= \underline{481 \text{ with 1 d.f.}}
 \end{aligned}$$

7. Feed sum of squares corrected ( $C_{cc}$ )

$$\begin{aligned}
 &= \frac{\sum_{k=1}^t \left( \sum_{i=1}^r \sum_{j=1}^s \sum_{l=1}^q x_{ijkl} \right)^2}{rsq} - C, \text{ with } (t-1) \text{ d.f.} \\
 &= \frac{681^2 + 749^2}{16} - 63,903 \\
 &= 64,048 - 63,903 \\
 &= \underline{145, \text{ with 1 d.f.}}
 \end{aligned}$$

8. The computation of the type of milling  $\times$  speed interaction sum of squares corrected ( $AB_{cc}$ ) is based upon Table 8 of type of milling—speed sums, with  $(r-1)(s-1)$  d.f. obtained from Table 7.

Table 8. Type of milling  $\times$  speed.

Speed	Type of milling		Sum
	Up	Down	
$s_1$	187	590	777
$s_2$	<u>173</u>	<u>475</u>	<u>653</u>
Sum	365	1065	1430

Subclasses sum of squares corrected

$$\begin{aligned}
 & \sum_{i=1}^r \sum_{j=1}^s \left( \sum_{k=1}^t \sum_{l=1}^q x_{ijkl} \right)^2 \\
 &= \frac{\sum_{i=1}^r \sum_{j=1}^s \left( \sum_{k=1}^t \sum_{l=1}^q x_{ijkl} \right)^2}{tq} - c \\
 &= \frac{187^2 + 590^2 + 178^2 + 475^2}{8} - 63,903 \\
 &= 80,047 - 63,903 = 16,144
 \end{aligned}$$

∴ Type of milling x speed interaction sum of squares corrected

$$\begin{aligned}
 &= 16,144 - (\text{speed sum of squares corrected} + \text{type of milling sum of squares corrected}) \\
 &= 16,144 - (481 + 15,313)
 \end{aligned}$$

$$AB_{cc} = 350, \text{ with 1 d.f.}$$

9. The computation of the type of milling x rate of feed interaction sum of squares corrected ( $AC_{cc}$ ) is based upon Table 9 with  $(r-1)(t-1)$  degrees of freedom. This table is obtained from Table 7.

Table 9. Type of milling x feed

Table feed	Type of milling		Sum
	Up	Down	
$F_1$	146	535	681
$F_2$	219	530	749
Sum	365	1065	1430

Subclasses sum of squares corrected

$$\begin{aligned}
 & \sum_{i=1}^r \sum_{k=1}^t \left( \sum_{j=1}^s \sum_{l=1}^q x_{ijkl} \right)^2 \\
 &= \frac{\sum_{i=1}^r \sum_{k=1}^t \left( \sum_{j=1}^s \sum_{l=1}^q x_{ijkl} \right)^2}{sq} - c \\
 &= \frac{146^2 + 535^2 + 219^2 + 530^2}{8} - 63,903 \\
 &= 79,550 - 63,903 \\
 &= 15,647
 \end{aligned}$$

$$\begin{aligned}
 & \therefore \text{Type of milling } \times \text{feed interaction sum of squares corrected} \\
 & = 15,647 - (\text{type of milling sum of squares corrected} + \text{feed sum} \\
 & \quad \text{of squares corrected}) \\
 & = 15,647 - (15,313 + 145) \\
 & \text{AC}_{\text{cc}} = \underline{189}, \text{ with 1 d.f.}
 \end{aligned}$$

10. Feed  $\times$  speed interaction sum of squares corrected ( $\text{BC}_{\text{cc}}$ ) is computed similarly by means of feed  $\times$  speed table of sums with  $(s-1)(t-1)$  d.f. Table 10 is obtained from Table 7.

Table 10. Speed  $\times$  feed.

Feed	Speed		Sum
	$S_1$	$S_2$	
$F_1$	348	333	681
$F_2$	429	320	<u>749</u>
Sum	777	653	1430

Subclasses sum of squares corrected

$$\begin{aligned}
 & \sum_{j=1}^s \sum_{k=1}^t \left( \sum_{i=1}^r \sum_{l=1}^q x_{ijkl} \right)^2 \\
 & = \frac{\sum_{j=1}^s \sum_{k=1}^t \left( \sum_{i=1}^r \sum_{l=1}^q x_{ijkl} \right)^2}{rq} - c \\
 & = \frac{348^2 + 429^2 + 333^2 + 320^2}{8} - 63,903 \\
 & = 64,804 - 63,903 \\
 & = 901
 \end{aligned}$$

$$\begin{aligned}
 & \therefore \text{Feed } \times \text{speed interaction sum of squares corrected} \\
 & = 901 - (\text{speed sum of squares corrected} + \text{feed sum of squares} \\
 & \quad \text{corrected}) \\
 & = 901 - (481 + 145) \\
 & \text{BC}_{\text{cc}} = \underline{275}, \text{ with 1 d.f.}
 \end{aligned}$$

11. The three factor interaction i.e., type of milling x speed, feed, sum of squares corrected ( $ABC_{cc}$ ) is computed as follows with  $(r-1)(s-1)(t-1)$  degrees of freedom:

$$\begin{aligned} \text{Subclasses sum of squares corrected} &= (A_{cc} + B_{cc} + C_{cc} + AB_{cc} + AC_{cc} + BC_{cc}) \\ &= 16,796 - (15,313 + 481 + 145 + 350 + 189 + 275) \\ &= 43 \end{aligned}$$

$$\therefore \text{Type of milling} \times \text{speed} \times \text{feed interaction} = 43, \text{ with 1 d.f.}$$

$$\text{sum of squares corrected (}ABC_{cc}\text{)}$$

Table 11 gives the general analysis of variance for the three factor factorial in randomized complete block design.

Table 12 gives the computed calculation of the analysis of variance for the surface finish data.

The test of the significance on the variables is performed by dividing each of the variable mean squares by the experimental error mean square and comparing the values for significance with the theoretical variance ratio tabulated in Snedecor's table of variance ratio. The level of the significance is maintained at 5 percent.

#### RESULTS AND DISCUSSION

Table 12 shows the results of the analysis of variance for the surface finish data. It was concluded from the analysis that all the three main effects (factors) i.e., type of milling, speed and feed, and their two factor interactions were significant. The highest order interaction i.e., the type of milling  $\times$  speed  $\times$  feed was found to be nonsignificant, therefore, it was concluded that type of milling, speed and feed are independent of surface finish. It was also concluded from the analysis that the work piece was of homogeneous nature because the replicate mean square was nonsignificant.

Table 11. General analysis of variance with E.M.S. for  $2^3$  factorial randomized complete block design.

Source of variation	Degrees of freedom (d.f.)	Mean square	Expected mean square E.M.S.
Replicate	(q-1)	$R_{qq}/(q-1)$	$\sigma^2 + \sigma_k^2$
FACTORS			
Type of milling	(r-1)	$A_{qq}/(r-1)$	$\sigma^2 + stq \sum_{i=1}^r A_{1i}^2/(r-1)$
Cutting speed	(s-1)	$B_{qq}/(s-1)$	$\sigma^2 + rtq \sum_{j=1}^s B_{1j}^2/(s-1)$
Rate of feed	(t-1)	$C_{qq}/(t-1)$	$\sigma^2 + rq \sum_{k=1}^t C_{1k}^2/(t-1)$
INTERACTION			
Type of milling $\times$ speed	(r-1)(s-1)	$AB_{qq}/(r-1)(s-1)$	$\sigma^2 + tq \sum_{i=1}^r AB_{1i}^2/(r-1)(s-1)$
Type of milling $\times$ feed	(r-1)(t-1)	$AC_{qq}/(r-1)(t-1)$	$\sigma^2 + sq \sum_{j=1}^s AC_{1j}^2/(r-1)(t-1)$
Speed $\times$ feed	(s-1)(t-1)	$BC_{qq}/(s-1)(t-1)$	$\sigma^2 + rq \sum_{k=1}^t BC_{1k}^2/(s-1)(t-1)$
Milling $\times$ speed $\times$ feed	(r-1)(s-1)(t-1)	$ABC_{qq}/(r-1)(s-1)(t-1)$	$\sigma^2 + q \sum_{i=1}^r \sum_{j=1}^s \sum_{k=1}^t ABC_{1ijk}^2/(r-1)(s-1)(t-1)$
RESIDUAL			
Experimental error	(q-1)(rst-1)	$E_{qq}/(q-1)(rst-1)$	$\sigma^2$
Total	(rstq-1)	$T_{qq}/(rstq-1)$	

Table 12. Analysis of variance for the effects of factors (type of milling, speed and feed) on the surface finish in the machining of mild steel for  $2^3$  factorial randomized complete block design.

Source of variation	d.f.	Mean square	Critical F values at .05	Calculated F values
Replicates	3	5.33	3.07	
<b>FACTORS</b>				
Type of milling	1	15,313	4.32	1302.000*
Cutting speed	1	481	4.32	40.900*
Rate of feed	1	145	4.32	12.330*
<b>INTERACTION</b>				
Milling x speed	1	350	4.32	29.800*
Milling x feed	1	189	4.32	16.100*
Speed x feed	1	275	4.32	23.400*
Milling x speed x feed	1	43	4.32	3.660
<b>RESIDUAL</b>				
Experimental error	21	11.76		

\*Significant at .05 level.

Since the two factor interactions were significant namely, type of milling x speed, type of milling x feed and speed x feed, it would be appropriate to derive more information about the interaction effects.

### Two Factor Interaction

The two factor interaction being significant, it could be concluded that there is a failure of the levels of one factor to retain the same order and magnitude of performance (within random sampling error) throughout all levels of the second factor. This could be readily seen in Figures 5, 6 and 7 obtained from Tables 13, 14 and 15 respectively which are the means of the two factor interaction tables (Tables 13, 14 and 15 are obtained after dividing the cell values of Tables 3, 9 and 10 by 8). It is concluded from the analysis and from Figures 5 and 6 that the effects of the type of milling changes with changing speeds and feeds, and that the effects of the feeds changes with changing speeds as per the analysis and Figure 7.

Table 13. Mean of factor interaction.

Speed	Type of mill x speed	
	Up	Down
$S_1$	23.40	73.80
$S_2$	22.26	59.40

Table 14. Mean of factor interaction.

Feed	Type of mill x feed	
	Up	Down
$F_1$	18.26	66.90
$F_2$	27.40	66.30

Table 15. Mean of factor interaction.

Feed	Feed x speed	
	$S_1$	$S_2$
$F_1$	43.50	41.60
$F_2$	53.60	40.00

It would be important to investigate some of the facts concerning the type of milling on surface finish data. Table 16a and b shows the means of four replicates for speed and feed under up milling and down milling. Figure 8 represents the interaction effects.

Table 16. Mean of four replicates for speed and feed under up and down milling.

Feed	a.	Up		b.	Down	
		$S_1$	$S_2$		$S_1$	$S_2$
$F_1$	17	19.5		70	63.75	
$F_2$	29.75	25		77.5	55	

As the objective of this experiment was to determine which type of milling, either up or down, under different combinations of machining conditions would give a better quality of surface finish, it is now concluded that up milling gives a better quality of surface finish than down milling with the given combinations of speeds and feed, as per analysis and Figure 8, or it may be also stated that the effects of the type of milling changes with changing speeds and feeds, but not to that extent that down milling would give a better quality of surface finish than up milling, working with the combinations of the selected machining conditions i.e., speeds and feeds, and holding other factors constant.

Material: Mild Steel SAE 1018

Cutter: High Speed Steel  
 4" Diameter,  $\frac{1}{2}$ " Width  
 22 teeth,  $30^\circ$  clearance angle  
 $10^\circ$  rake angle

Depth of Cut: 0.03"

$S_1 = 79$  ft. per minute

$S_2 = 126$  ft. per minute

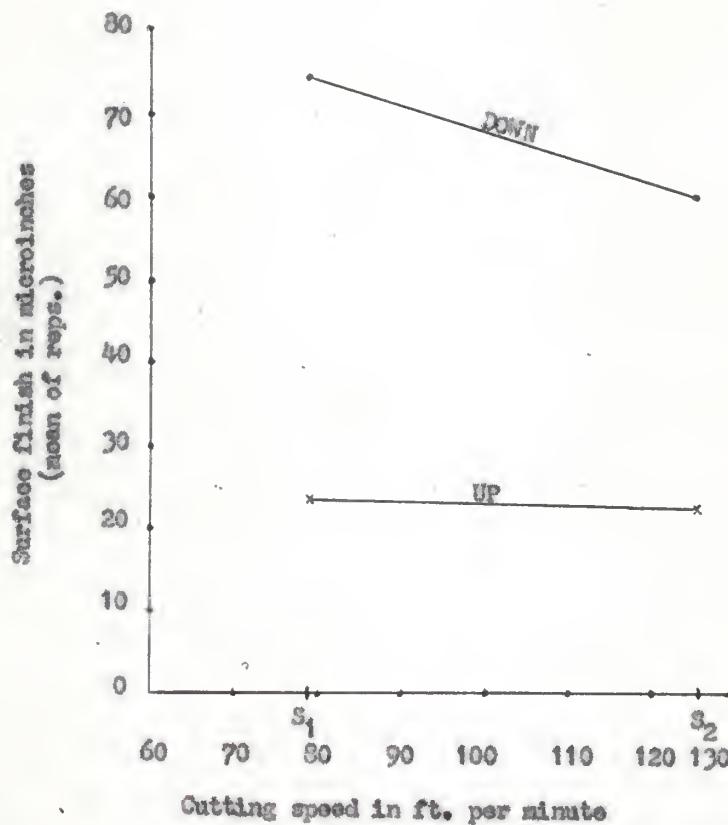


Figure 5. Cutting speed vs. surface finish for two types of milling.

Material: Mild Steel SAE 1018

Cutter: High Speed Steel  
 4" Diameter,  $\frac{1}{2}$ " Width  
 22 teeth,  $3^\circ$  clearance angle  
 $10^\circ$  Rake angle

Depth of Cut: 0.03"

$F_1 = 1\frac{1}{2}$  inches per minute

$F_2 = 3\frac{1}{2}$  inches per minute

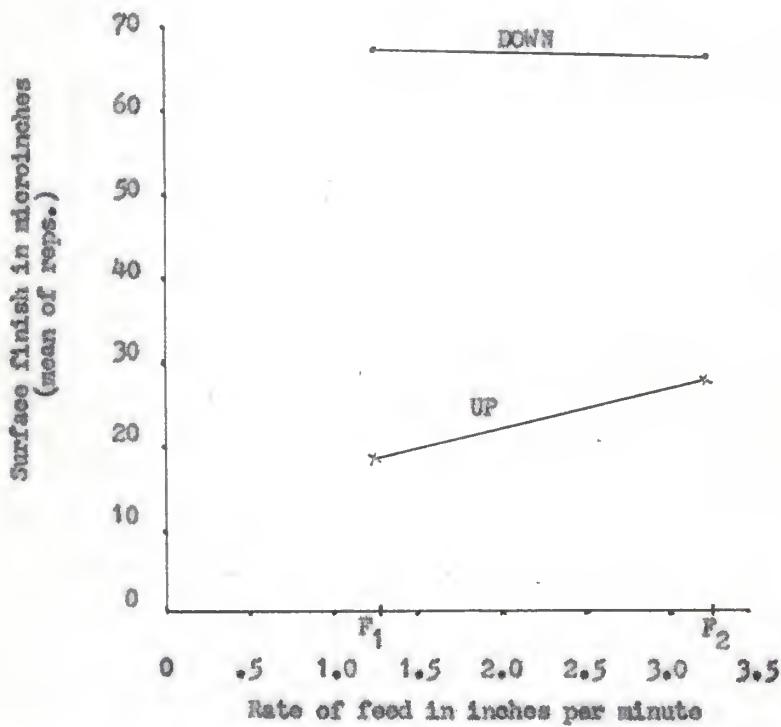


Figure 6. Rate of feed vs. surface finish for two types of milling.

Material: Mild Steel SAE 1018

Cutter: High Speed Steel  
 4" Diameter,  $\frac{1}{2}$ " Width  
 22 teeth,  $30^\circ$  clearance angle  
 $10^\circ$  Rake angle

Depth of Cut: 0.03"

$F_1 = 1\frac{1}{2}$ "/minute  $S_1 = 79$  ft./minute

$F_2 = 3\frac{1}{2}$ "/minute  $S_2 = 126$  ft./minute

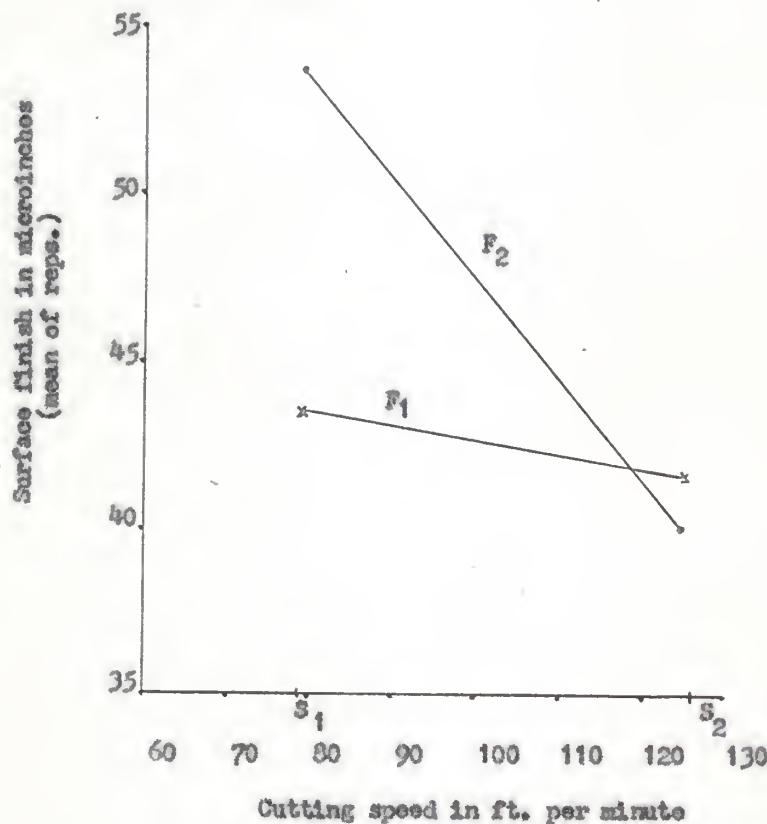


Figure 7. Cutting speed vs. surface finish for various feeds.

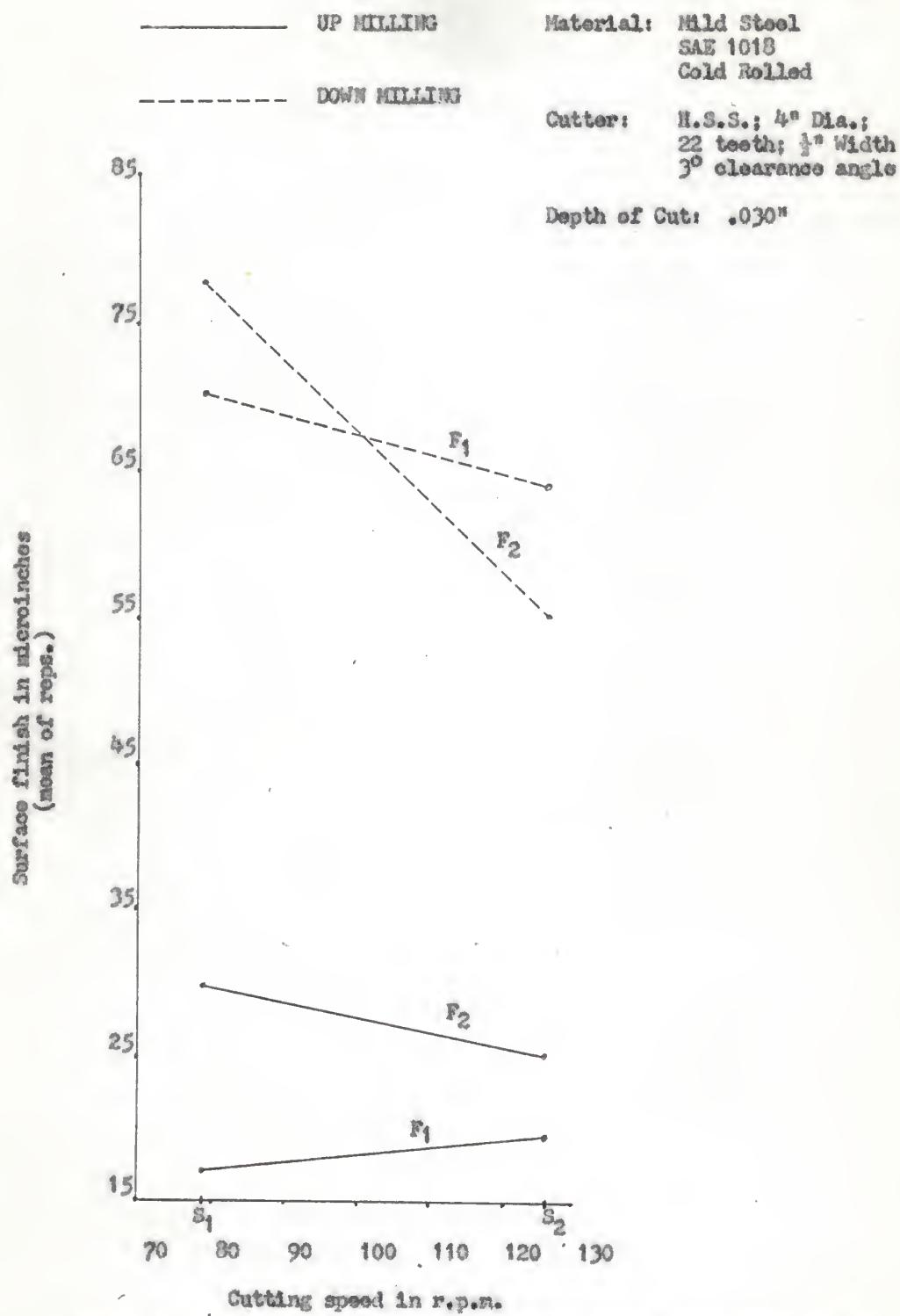


Figure 8. Cutting speed vs. surface finish for various feeds under up milling and down milling.

Figure 8 also shows that a speed of 79 f.p.m. and a feed of  $1\frac{1}{2}$  inches per minute appears to be the machining condition most desired under up milling to get a better quality of surface finish.

Further study was made on the surface finish data by using one of the multiple comparison methods.

#### Multiple Comparison Test for Means

When an F-test has led to the rejection of null hypothesis ( $H_0$ ), one is led to believe that some of the population means are unequal. There remains a question: which population means are equal and which are not. To test which means are equal and which are not, multiple comparison method is used.

#### L S D Test

One commonly used procedure (11) for comparing the differences among a set of means and for comparing each one of a set of means with a standard treatment is the least significant difference, or the L S D test. The L S D is equal to the product of the standard error of the mean, the  $\sqrt{\frac{2}{f}}$ , and the value of "t" at the 5 percent level (for this experiment) for the number of degrees of freedom,  $f$ , associated with the standard error of the mean.

Symbolically the L S D is  $t_{\alpha/2} \cdot \text{Error M.S.} \cdot \sqrt{\frac{2}{n}}$ .

where: M.S. stands for mean square

d.f. stands for degrees of freedom

n stands for number of replicates.

Differences between all two mean comparisons are compared with L S D, and if the differences exceeds the L S D the means are said to come from population of different means.

Tables 13, 14 and 15 give the means of all the two factor interaction and Tables 17, 18 and 19 give the ordered array of sample means.

The level of significance is maintained at .05 level.

The difference between all two means will be compared with:

$$t_{\alpha}, \text{ Error M.S. d.f.} \quad \boxed{\frac{2 \text{ Error M.S.}}{n}}$$

where:  $t_{\alpha}, \text{ Error M.S. d.f.} = 2.08$   $\left\{ \text{Table of distribution of } \chi^2(12) \right\}$   
 $\text{Error M.S.} = 11.76$

$$n = 8$$

$$\therefore \text{L S D} = 2.08 \sqrt{\frac{2 \times 11.76}{8}} = 3.564$$

Table 16 shows the speed does not have significant effect on up milling but it does in down milling i.e., increasing or decreasing the speed from 79 ft. per minute to 126 ft. per minute does not have significant effect in the quality of surface finish in up milling but the combination of high speed with down milling gives a better quality of surface finish when compared with low speed.

Table 17 shows feed does not have significant effect on down milling but it does on up milling.

Table 18 shows the combination of  $S_1 F_2$  gives a rough finish.

Table 17. Ordered array of sample means and indications of significant differences among the means of type of milling  $\times$  speed.

Means	22.26	23.40	59.40	73.80
L S D = 3.564	means lying above the horizontal line are not significantly different.			

Table 18. Ordered array of sample means and indications of significant differences among the means of type of milling x feed.

Means	18.26	27.40	<u>66.30</u>	66.90
L S D = 3.564	means lying above the horizontal line are not significantly different.			

Table 19. Ordered array of sample means and indications of significant differences among the means of speed x feed.

Means	40.0	41.6	43.5	53.6
L S D = 3.564	means lying above the horizontal line are not significantly different.			

#### CONCLUSION

It was inferred from this study that up milling gives a better quality of surface finish than down milling with the given combinations of speeds and feeds. It can also be stated that the effects of the type of milling changes with changing speeds and feeds, as per the analysis, but not to the extent that down milling would give a better quality of surface finish than up milling, working with the combinations of selected machining conditions.

It was also concluded that the combinations of low levels of speeds such as 79 feet per minute and low levels of feeds such as  $1\frac{1}{4}$  inches per minute appears to be the machining condition most desired under up milling to get a better quality of surface finish. The experiments also brought out the fact that speed does not have significant effect on up milling but does have an effect on down milling. That feed does not have significant effect on down milling but it does on up milling. That the combination of lower speeds such as 79 feet per minute and higher feeds such as  $3\frac{1}{2}$  inches per minute give a rough finish.

## ACKNOWLEDGMENTS

The author sincerely appreciates the guidance and inspiration of Dr. G. F. Schrader, Head of the Department, under whose direction this work was conducted.

Sincere appreciation is extended to Dr. L. F. Marcus, Assistant Professor of the Department of Statistics, for his very helpful suggestions in statistical analysis.

Thanks is expressed to Mr. C. L. Nelson, Instructor of Industrial Engineering, for his help in constructing the necessary equipment, and to other staff for their various assistances.

## REFERENCES

1. Federer, W. T. Experimental Design. The MacMillan Company, 1963, page 2.
2. Mennel, R. F. and E. A. Jeffery. Evaluation of several lathe tool life testing techniques. Collection of ASTE papers, 1959, page 216.
3. Snedecor, G. W. Statistical Methods. The Iowa State University Press, 1962, page 246.
4. A Treatise on Milling and Machines. The Cincinnati Milling Machine Company. Third Edition, 1951, page 238.
5. Federer, W. T. Experimental Design. The MacMillan Company, 1963, page 114.
6. \_\_\_\_\_\_. Experimental Design. The MacMillan Company, 1963, page 168.
7. Fisher, R. A. The Design of Experiments, Oliver and Boyd, 1935, Sec. 40.
8. Yates, F. The Design and Analysis of Factorial Experiments. Imperial Bureau of Soil Science, Harpenden, England, 1937, pages 1-95.
9. Fisher, R. A. The Design of Experiments. Oliver and Boyd, 1935, pages 22-26.
10. Cochran, G. W. and Cox, G. M. Experimental Design. John Wiley and Sons, Inc. Second Edition, 1957, page 577.
11. Federer, W. T. Experimental Design. The MacMillan Company, 1963, page 20.
12. Snedecor, G. W. Statistical Methods. The Iowa State University Press, 1962, page 46.

STATISTICAL ANALYSIS OF QUALITY OF SURFACE FINISH  
IN MILLING OPERATIONS

by

HUSAIN ALI HASHAM

L. M. E. (Mechanical Engineering), Government Polytechnic,  
Hyderabad, India, 1960  
B. S. (Industrial Engineering), Kansas State University, 1963

---

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the  
requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY  
Manhattan, Kansas

1964

The principal aim of this study was to determine which of the two types of milling, up or down, gives a better quality of surface finish under different machining conditions i.e., cutting speeds and rate of feeds, on cold rolled mild steel SAE 1018. In addition, it would also be determined as to what amount of variability in the quality of surface finish is present between these two types of milling.

Statistical methods were employed to investigate the effects on surface finish characteristics of types of milling with the combinations of different levels of cutting speeds and rate of feeds.

The experiment was performed with three factor factorial in a randomized complete block design, two types of milling, two levels of cutting speed and two levels of table feed. The number of replicates was chosen to be four. Other than changing the required types of milling, cutting speed and table feed care was taken to see that all relevant factors which may have an influence on surface finish were either held constant or minimized as far as practical.

The whole experiment was performed with a high speed steel cutter of 4" in diameter, 22 teeth,  $\frac{1}{2}$ " width,  $3^{\circ}$  clearance angle and  $10^{\circ}$  rake angle.

Trial experiments were conducted to determine the levels of cutting speed and rate of feed with different depth of cuts and tooth clearance angle. It was then decided to work with 79 feet per minute and 126 feet per minute as the two levels of cutting speed, and  $1\frac{1}{4}$  inches per minute and  $3\frac{1}{4}$  inches per minute as the two levels of rate of feed. It was also decided to use 0.030 inches as the depth of cut, and  $3^{\circ}$  tooth clearance angle.

The measurement of surface finish was accomplished by means of a profilometer. All machining operations were performed dry, but air blast was applied for a check to see the chips do not stick to the cutter. The level of significance was maintained at 5 percent.

The investigations showed that all the three main effects i.e., type of milling, speed and feed, and their two factor interactions were significant. The three factor interaction i.e., the type of milling  $\times$  speed  $\times$  feed was found to be nonsignificant. It was also concluded that the work piece was of homogeneous nature because the replicate mean square was nonsignificant.

The two factor interaction being significant it could be concluded that the effects of the type of milling changes with changing speeds and feeds, and that the effects of the speed changes with changing feeds.

As the objective of this study was to determine which type of milling, either up or down, under different combinations of machining conditions would give a better quality of surface finish, it is now concluded that the effects of the type of milling changes with changing speeds and feeds, but not to that extent that down milling would give a better quality of surface finish than up milling working with the combinations of selected machining conditions. It was also noticed that a speed of 79 feet per minute and a feed of  $1\frac{1}{4}$  inches per minute appears to be the machining condition most desired under up milling.

Further study was made on surface finish data by using one of the multiple comparison methods. Least significant difference (LSD), one commonly used procedure for comparing the differences among a set of means, disclosed the fact that speed does not have a significant effect on up milling but it does in down milling. It was also known that feed does not have a significant effect on down milling but it does on up milling, and also that the combination of 79 feet per minute of speed with  $3\frac{1}{4}$  inches per minute of feed gives a rough finish.